Math 20550 - Summer 2016
Double Integrals
June 29, 2016

Problem 1. *Compute* $\int_{-1}^{1}\int_{-2}^{2}(x^2 - y^2)dydx$.

Problem 2. *Compute the integral* $\int_0^{\pi}\int_0^{\sin x}(1+\cos x)dydx$.

Problem 3. *Find the area bounded by the curves* $y = 4 - x^2$ *and* $y = x + 2$.

Problem 4. *Switch the order of integration in the integral* $\int_1^{10}\int_0^{\ln y} f(x,y)dxdy$.

Problem 5. *Switch the order of integration to combine the two integrals into one, then compute the integral:*

$$\int_0^2\int_0^x dydx + \int_2^4\int_0^{4-x} dydx.$$

Problem 6. *Compute the integral* $\int_0^4\int_{\sqrt{x}}^2 \frac{3}{2+y^3}dydx$.

Problem 7. *Compute the integral* $\int_0^2\int_x^2 x\sqrt{1+y^3}dydx$.

Problem 8. *Compute the integral* $\int_0^3\int_0^{\infty} \frac{x^2}{1+y^2}dydx$.

Problem 9. *Find the volume under the surface* $z = 4 - y^2$ *which lies above the region* R *in the* xy*-plane which is bounded by the* y*-axis,* $y = 2$*, and* $y = x$.

Problem 10. *Find the volume of the solid bounded by the* xy*-plane, the planes* $y = 1$, $x = 0$, $y = x$, *and the surface* $z = 1 - xy$.

Problem 11. *Set up a double integral which computes the volume bounded by the paraboloids* $z = 1 - x^2 - y^2$ *and* $z = x^2 + y^2 - 1$.

Problem 12. *Give a geometric argument to show that*

$$\int_0^3\int_0^{\sqrt{9-y^2}} \sqrt{9-x^2-y^2}dxdy = \frac{9\pi}{2}.$$

Problem 13 (Challenge?)**.** *Determine the region* R *which minimizes the value of the integral*

$$\iint_R (x^2+y^2-4)dA.$$